SU(N) Quantum Hall Skyrmions

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We have investigated skyrmions in N-component quantum Hall systems. We find that SU(N) skyrmions are the lowest energy charged excitations for filling factors $\nu = 1, 2, \ldots, N-1$ for small enough symmetry breaking terms. $N > 2$ skyrmions can be realized in Si QH systems based on the (110) or (111) interfaces of Si, or perhaps in Si (100) systems, where the spin and valley isospin together provide an SU(4)-symmetry, or in multilayer QH systems. We also present Hartree-Fock results for a phenomenological easy-axis SU(2)-breaking model appropriate to valley degeneracy.

Textured quasiparticle excitations in the quantum Hall (QH) effect involving nontrivial configurations of electron spin ("skyrmions") were proposed in [1]. There is now strong experimental evidence that such excitations are the lowest energy charged quasiparticles at filling fraction $\nu = 1$ [2–4]. The physics behind these excitations is as follows. First, even in the absence of Zeeman coupling, the QHE system is an exchange ferromagnet, owing to the degeneracy of the Landau levels. Second, spin textures carry charge; the charge density is proportional to the skyrmion (topological) density. Finally, there is a competition between Coulomb energy, which wants to make skyrmions of infinite size, and the Zeeman energy, which wants to shrink the skyrmions to a point; this determines the size of the skyrmion. (While such topological excitations always exist, they are not necessarily the lowest energy charged excitations [5].) These textures exist also in other two-component QHE systems, such as quantum Hall bilayers, in which the layer index plays the role of a pseudospin [6].

Here we study skyrmions in multi-component QH systems [7]. We consider the case where there are N degenerate internal "orbitals" for each electron, leading to an SU(N) symmetry. Our principal motivation for this generalization is Si QH systems, in which, in addition to spin, there is a valley degeneracy for each single particle state [8]. The valley degeneracy depends on the orientation of the interface along which the two-dimensional electron gas lives. For a (100) interface, which is the orientation usually used in Si MOSFET QH systems, the six valleys are split into an upper quadruplet and a lower doublet. The doublet is then further split by the interfacial electric field E, analogous to the Zeeman splitting of up and down spin states in a magnetic field. However, for a (110) interface the quadruplet lies lower and is split into two doublets by E, and in the case of a (111) interface all six valleys remain degenerate. With $N_{\rm v}$ degenerate valleys and two spin states, the appropriate symmetry group, in the absence of Zeeman and other symmetry-breaking terms, is SU(N) where $N = 2N_{\rm v}$.

Our main results are as follows. We derive the energy functional for SU(N) QH skyrmions at filling factor $\nu = 1, 2, \ldots, N-1$ and show that the energy required to create a skyrmion-antiskyrmion pair is half the energy to create a (polarized) quasielectron-quasihole pair. In addition to their charge, the skyrmions are characterized by $N-1$ quantum numbers. For Si (100) QH systems for example, this implies that for small enough symmetry-breaking terms SU(4) skyrmions would be the lowest energy charged excitations at $\nu = 1, 2, 3$. However, this limit is hard to reach. Better candidates for SU(4) and SU(6) skyrmions are Si (110) and Si (111) QH systems respectively. Multilayer QH systems offers another possibility to realize $N > 2$ skyrmions. We consider briefly symmetry breaking terms and determine the energies for valley SU(2) skyrmions as a function of the strength of an easy axis symmetry breaking term.

SU(N) QH Skyrmions – We first generalize the microscopic derivation of the skyrmion energy and charge density of Yang *et al.* [6] to the case of N electron "flavors". This will allow us to map the low-energy dynamics onto a nonlinear sigma model. Let $\mu = 1, \ldots, N_\phi$ index the degenerate lowest Landau level single particle states on the plane; N_ϕ is the total number of flux quanta. Consider now the Slater determinant state $|\Psi_m[0]\rangle = \prod_{a=1}^{m} \prod_{\mu=1}^{N_\phi} \psi_{\mu a}^\dagger |0\rangle$ of integer filling factor $\nu = m \le N$. We define the *first*-quantized operators $S_{\alpha\beta}(i)$ in terms of their action in a flavor-diagonal basis for a given electron i: $S_{\alpha\beta}(i)$ annihilates any state in which the flavor of electron i is not β and otherwise changes the flavor from β to α. Here $i \in \{1, \ldots, \mathcal{N}\}$, where $\mathcal{N} = mN_\phi$ is the total electron number. These operators satisfy SU(N) commutation relations $[S_{\alpha\beta}, S_{\rho\lambda}] = \delta_{\alpha\lambda} S_{\rho\beta} - \delta_{\rho\beta} S_{\alpha\lambda}$ for each electron and commute for different electrons. Note that Ψ_m is invariant under the U(m)$\times$ U($N-m$) subgroup generated by $\mathcal{S}_{ab}$ and $\mathcal{S}_{\bar{a}\bar{b}}$, where $a, b \in \{1, \ldots, m\}$, $\bar{a}, \bar{b} \in \{m+1, \ldots, N\}$, and $\mathcal{S}_{\alpha\beta} = \sum_i S_{\alpha\beta}(i)$. Following [6] and [9], we define the flavor texture

$$|\Psi_m[q]\rangle = \exp \int d^2r \left(q_{a\bar{a}}(\boldsymbol{r}) \overline{S_{\bar{a}a}(\boldsymbol{r})} - q_{a\bar{a}}^*(\boldsymbol{r}) \overline{S_{a\bar{a}}(\boldsymbol{r})} \right) |\Psi_m[0]\rangle$$

$$= \exp \int \frac{d^2k}{(2\pi)^2} \sum_{i=1}^{\mathcal{N}} \Big(\hat{q}_{a\bar{a}}(\boldsymbol{k}) \, S_{\bar{a}a}(i) \\ - \hat{q}^*_{a\bar{a}}(-\boldsymbol{k}) \, S_{a\bar{a}}(i) \Big) \times \overline{\rho}_i(-\boldsymbol{k}) \, | \, \Psi_m[0] \, \rangle$$

where $q_{a\bar{a}}(\boldsymbol{r})$ is a complex $m \times (N - m)$ matrix-valued function of $\boldsymbol{r}$ and $\hat{q}_{a\bar{a}}(\boldsymbol{k})$ is its Fourier transform,

$$S_{\alpha\beta}(\boldsymbol{r}) = \sum_{i=1}^{\mathcal{N}} S_{\alpha\beta}(i) \, \delta(\boldsymbol{r} - \boldsymbol{r}_i) , \tag{2}$$

and $\overline{\mathcal{O}}$ denotes the lowest Landau level projection of $\mathcal{O}$. The projected density operator satisfies

$$\overline{\exp(-i\boldsymbol{k} \cdot \boldsymbol{r}_i)} = \overline{\rho}_i(\boldsymbol{k})$$
$$\overline{\rho}_i(\boldsymbol{k}) \, \overline{\rho}_i(\boldsymbol{p}) = \exp(\tfrac{1}{2}\boldsymbol{k} \cdot \boldsymbol{p} \, \ell^2 + \tfrac{i}{2} \cdot \boldsymbol{k} \wedge \boldsymbol{p} \, \ell^2) \overline{\rho}_i(\boldsymbol{k} + \boldsymbol{p}) \tag{3}$$

with $\ell = \sqrt{\hbar c / eB}$ the magnetic length and $\boldsymbol{k} \wedge \boldsymbol{p} \equiv \hat{\boldsymbol{z}} \cdot \boldsymbol{k} \times \boldsymbol{p}$. The ground state is homogeneous, hence

$$\overline{\rho}(\boldsymbol{k}) \, | \, \Psi_m[0] \, \rangle = \delta(\boldsymbol{k}) \mathcal{N} \, | \, \Psi_m[0] \, \rangle , \tag{4}$$

where $\overline{\rho}(\boldsymbol{k}) = \sum_i \overline{\rho}_i(\boldsymbol{k})$ is the total projected density operator. The $q_{a\bar{a}}$ are complex coordinates parametrizing the coset space $U(N)/[U(m) \times U(N - m)]$, which is the target space of the sigma model. Since Π_2 of the target space is Z, topological excitations - skyrmions - characterised by *one* integer topological charge exist for any N and m. For $m = 1$ the target space is CP^{N-1}, which for $N = 2$ is S^2.

Following [6] we calculate the excess charge density and the energy in the flavor texture state in a gradient expansion of q:

$$\delta\rho(\boldsymbol{r}) = \langle \, \Psi_m[q] \, | \, \overline{\rho(\boldsymbol{r})} \, | \, \Psi_m[q] \, \rangle - \langle \, \Psi_m[0] \, | \, \overline{\rho(\boldsymbol{r})} \, | \, \Psi_m[0] \, \rangle$$
$$= \frac{i}{2\pi} \nabla q_{a\bar{a}} \wedge \nabla q^*_{a\bar{a}} + \dots \tag{5}$$
$$E = \langle \, \Psi_m[q] \, | \, \overline{V} \, | \, \Psi_m[q] \, \rangle - \langle \, \Psi_m[0] \, | \, \overline{V} \, | \, \Psi_m[0] \, \rangle$$
$$= 2\rho^\circ_s \int d^2r \, |\nabla q_{a\bar{a}}|^2 + \dots \tag{6}$$

where

$$\overline{V} = \tfrac{1}{2} \int \frac{d^2k}{(2\pi)^2} \, \hat{v}(\boldsymbol{k}) \, \overline{\rho}(-\boldsymbol{k}) \, \overline{\rho}(\boldsymbol{k}) \tag{7}$$

$$\rho^\circ_s = -\frac{1}{32\pi^2} \int_0^\infty dk \, k^3 \, h(k) \, \hat{v}(k) \tag{8}$$

$$h(k) = \frac{1}{\mathcal{N}} \langle \, \Psi_m[0] \, | \, \overline{\rho}(-\boldsymbol{k}) \, \overline{\rho}(\boldsymbol{k}) \, | \, \Psi_m[0] \, \rangle .$$

Here $\hat{v}(q)$ is the Fourier transform of the interparticle potential. For Coulomb interactions $\rho^\circ_s = e^2/(16\sqrt{2\pi}\epsilon\ell)$.

To go beyond the gradient expansion is a tedious proposition. However, we may recognize the above results as those obtained from expanding the energy E and topological density J^0 of the $U(N)/[U(m) \times U(N - m)]$ sigma model. A general element in the coset space may be written as $R = U^\dagger \Lambda U$, where $U = \exp \begin{pmatrix} 0 & q \\ -q^\dagger & 0 \end{pmatrix}$ and $\Lambda = \begin{pmatrix} 1_m & 0 \\ 0 & -1_{N-m} \end{pmatrix}$ (q is the matrix $q_{a\bar{a}}$ and and 1_p is the p-dimensional unit matrix). The topological current is written

$$J^\mu = \frac{i}{16\pi} \, \epsilon^{\mu\nu\lambda} \, \text{Tr}(R \, \partial_\nu R \, \partial_\lambda R) ; \tag{9}$$

the total topological charge, $Q = \int d^2r \, J^0$, is an integer. The energy functional is given by

$$E_0[R] = \tfrac{1}{4}\rho^\circ_s \int d^2r \, \text{Tr}(\nabla R)^2 . \tag{10}$$

Our results are consistent with these, upon identifying $\delta\rho = J^0$, to lowest order in the gradient expansion. As shown by [6], the long-ranged Coulomb interaction appears at higher order, and a more accurate energy functional is therefore

$$E[R] = \tfrac{1}{4}\rho^\circ_s \int d^2r \, \text{Tr}(\nabla R)^2 \\ + \tfrac{1}{2} \int d^2r \int d^2r' \, J^0(\boldsymbol{r}) \, v(\boldsymbol{r} - \boldsymbol{r}') \, J^0(\boldsymbol{r}') . \tag{11}$$

The dynamics are determined by the kinetic contribution

$$\langle \, \Psi_m[q] \, | \, i\frac{\partial}{\partial t} \, | \, \Psi_m[q] \, \rangle = \frac{i}{2\pi\ell^2} \int d^2r \, q^*_{a\bar{a}} \, \partial_t q_{a\bar{a}} + \dots \tag{12}$$
$$\simeq \frac{i}{8\pi\ell^2} \int d^2r \int_0^1 du \, \text{Tr} \, (R \, \partial_u R \, \partial_t R) ,$$

where $R(u = 0, t) = \Lambda$ and $R(u = 1, t) \equiv R(t)$. The last line of the above equation is the appropriate generalization of the $m = 1$ nonlinear sigma model kinetic term (see [9]).

Skyrmion and quasiparticle energies – We now consider the energies of skyrmions. From $\text{Tr} \, |\partial_\mu R \pm i \, \epsilon_{\mu\nu} R \, \partial_\nu R|^2 \geq 0$ we find $E_0 \geq 4\pi\rho^\circ_s |Q|$. The minimum energy for a skyrmion (or antiskyrmion) with charge $Q = \pm 1$ (electric charge $\pm e$) at $\nu = m$ is thus $E_{\text{sk}} = 4\pi\rho^\circ_s = \tfrac{1}{4}\sqrt{\tfrac{\pi}{2}}(e^2/\epsilon\ell)$ (assuming Coulomb interactions). Comparing this to the energies of a polarized quasielectron, $E_e = 0$, and a polarized quasihole, $E_h = \sqrt{\tfrac{\pi}{2}}(e^2/\epsilon\ell)$, we see that, as in the SU(2) case, the gap to creating a skyrmion-antiskyrmion pair is half the gap to creating a pair of polarized quasiparticles. Note that the energies are independent of N and of m. We conclude that, for small enough symmetry breaking terms, skyrmions will be the lowest energy quasiparticles for $\nu = m = 1, 2, 3, \dots, N - 1$. Without the Coulomb term, the sigma model is scale invariant and skyrmions of any size with energy $E = 4\pi\rho^\circ_s |Q|$ exist. Coulomb interactions make the skyrmions infinitely large (the energy being the same). Symmetry breaking terms imply a cost to texturing spins, hence they favor small skyrmions. The energy is then larger than the lower

bound and the size of the skyrmion is determined by a competition between the Coulomb interaction and the symmetry breaking terms.

In addition to charge, the SU(2) skyrmions are characterized by their spin, *i.e.*, by their $\mathcal{S}^z$ eigenvalue. For SU(N) there are $N-1$ commuting generators, hence these skyrmions are characterized by $N-1$ quantum numbers. Explicit symmetry breaking terms may of course reduce the number of conserved quantities.

Experimental Realizations – We have shown that SU(N) skyrmions are the lowest energy quasiparticles in any N-component QH system provided the symmetry breaking terms are small enough. In the ordinary Si MOSFET QH systems, based on the (100) interface, the combination of spin and valley degeneracy leads, as pointed out above, to an SU(4) symmetry. However, it is important to note that the Zeeman term that breaks the SU(2)$_{\text{spin}}$ subgroup under normal conditions is too large for spin skyrmions to be the lowest energy charged excitations. If $\tilde{g} = g\mu_{\text{B}}B/(e^2/\epsilon\ell) > 0.054$, then the lowest energy quasiparticles are fully polarized [1]. In GaAs, this criterion is not met until $B \approx 25$T, but in Si, where $g \simeq 2$, the critical field is much lower: $B_{\text{c}} \approx 1.5$T. Thus, under normal conditions, the quasiparticles are fully spin polarized throughout the quantum Hall regime and one may not be able to reach the SU(4) skyrmion regime.

We believe the chances are best to realize $N > 2$ skyrmions in QH systems based on the (110) or (111) interfaces of Si. It is, for example, possible to grow high quality SiGe heterostructures of this type, and the symmetry breaking terms are believed to be small and tunable. This would give four and six degenerate valleys respectively and hence lead to SU(4) and SU(6) skyrmions. Multilayer QH systems, possibly with spin, offer another possibility to realize $N > 2$ skyrmions, if the symmetry breaking terms can be made small enough.

What happens in a particular system depends crucially on the type and size of the symmetry breaking terms. As an example of a possible scenario we discuss in some detail the (100) Si case, even though it may be hard to reach the SU(4) skyrmion regime in such systems.

SU(4) and (100) Si QH Systems – Ignoring symmetry breaking terms there are four degenerate lowest Landau levels and skyrmions are the lowest energy charged excitations at $\nu = 1, 2, 3$. If we denote the valley index by left (L) and right (R) and spin by up ($\uparrow$) and down ($\downarrow$), a basis of states is $\{\,|\,\text{L}\uparrow\,\rangle,\ |\,\text{L}\downarrow\,\rangle,\ |\,\text{R}\uparrow\,\rangle,\ |\,\text{R}\downarrow\,\rangle\,\}$, and the three commuting SU(4) generators can be choosen as $\sigma_{zR} = \text{diag}(0,0,1,-1)$, $\sigma_{zL} = \text{diag}(1,-1,0,0)$ and $\tau_{\uparrow} = \text{diag}(-1,0,1,0)$. $\sigma_{zR/L}$ measure the z-component of the spin in valley R/L, $\tau_{\uparrow}$ is the difference in the number of spin $\uparrow$ electrons in the R and L valleys.

We assume the most important symmetry breaking terms in Si are: the spin Zeeman term, $H_g = \frac{1}{2}g\mu_B B(\sigma_{zR} - \sigma_{zL})$, which affects only the SU(2)$_{\text{spin}}$ subgroup (and breaks it to U(1)) and a term H_w, which affects only the SU(2)$_{\text{valley}}$ subgroup and breaks it to a U(1) easy axis symmetry [10]. (A possible choice is

$H_w = w\sum_k(\tau_{\uparrow k} + \tau_{\downarrow k})(\tau_{\uparrow k+1} + \tau_{\downarrow k+1})$, $w \leq 0$ and k numbers the orbital states in the lowest Landau level. Another possibility is (15) below.) The Zeeman term causes the two spin $\downarrow$ lowest Landau levels to have lower energy than the two spin $\uparrow$ ones. Since H_w provides an easy axis anisotropy, the L/R valleys have the same energy. These symmetry breaking terms commute with the three diagonal generators σ_L, σ_R, $\tau_{\uparrow}$, which thus still give good quantum numbers.

Qualitatively, the behavior of the groundstate and the excitations as functions of the symmetry breaking parameters $\tilde{g}$ and w is as follows. For $\nu = 1$, the groundstate is two-fold degenerate: either all $L\downarrow$ or all $R\downarrow$ states are filled. At $\nu = 2$, all $L\downarrow$ and $R\downarrow$ states are filled and at $\nu = 3$, $L\uparrow$ or $R\uparrow$ are filled in addition. In each case, for small enough $\tilde{g}$ and $-w$ skyrmions are the lowest energy charged excitations. However, for large enough $\tilde{g}$ and/or $-w$ the skyrmion creation energy exceeds the energy to create polarized quasiparticles. Using Hartree-Fock, as for the $SU(2)$ case [11], it should be possible to obtain the energies and quantum numbers for the skyrmions as functions of $\tilde{g}, w$ and in particular to determine the range of parameters for which the skyrmions are the lowest energy charged excitations. For large $-w$, the problem reduces to the spin SU(2) case [1], whereas for large $\tilde{g}$ the problem instead reduces to the valley SU(2) case which we now discuss.

Silicon Valley SU(2) – If due to the largeness of $\tilde{g}$ the spins are fully polarized, valley-textured quasiparticles may be the lowest energy charged excitations in Si MOSFET quantum Hall systems. For Si (100) interfaces, the interface potential $v(z)$ leads to a splitting [12]

$$\Delta E = \left|\alpha\langle\frac{\partial v}{\partial z}\rangle\right| \tag{13}$$

of the lower doublet, where $\alpha \approx 0.23\,\text{Å}$, and where the average $\langle\partial v/\partial z\rangle$ is computed with respect to the envelope function $A(z)$, as described in ref. [12]. This is analogous to the usual Zeeman splitting, and the energy gap as a function of ΔE has been calculated previously [1]. Without recourse to techniques such as nuclear magnetic resonance, which has been used to detect spin skyrmions, it may be only through the energy gap, as measured in transport experiments through the activated behavior of ρ_{xx}, that valley skyrmions can be detected. Varying the interfacial potential $v(z)$ with an applied electric field or by pressure will lead to a change in ΔE and hence of the skyrmion-antiskyrmion creation energy.

Typically, though, the Si valley splitting ΔE is quite small – on the order of about 2 K – about 1% of the Coulomb energy $e^2/\epsilon\ell$ in Si at $B = 10$. It seems likely then that disorder-induced intervalley scattering will dominate this Zeeman term. This is described by a Hamiltonian of the form

$$\mathcal{H}' = \sum_{j=1}^{N_{\text{imp}}} \left[U_j\tau^+(\boldsymbol{r}_j) + U_j^*\tau^-(\boldsymbol{r}_j)\right] \tag{14}$$

where $\tau^{\pm}$ are the valley raising and lowering operators. The disorder thus enters as a random in-plane magnetic field which breaks $SU(2)_{valley}$ down to an easy axis $U(1)$ [10,13]. As we are primarily interested in the effects of this internal (rather than translational) symmetry breaking, we simplify the model by annealing the disorder in both space and (imaginary) time, and consider the following phenomenological Hamiltonian, which contains the correct symmetry breaking terms,

$$\mathcal{H} = \tfrac{1}{2}\int d^2r \int d^2r' \, V(\boldsymbol{r}-\boldsymbol{r}') : (\rho(\boldsymbol{r})-\rho_0)\,(\rho(\boldsymbol{r})-\rho_0):$$
$$+ \tfrac{1}{2}\int d^2r \int d^2r' \, W(\boldsymbol{r}-\boldsymbol{r}') : \tau^z(\boldsymbol{r})\,\tau^z(\boldsymbol{r}'): , \quad (15)$$

where

$$\rho(\boldsymbol{r}) = \sum_{\mu_1,\mu_2,\alpha} \varphi^*_{\mu_1}(\boldsymbol{r})\,\varphi_{\mu_2}(\boldsymbol{r})\,\psi^\dagger_{\mu_1\alpha}\psi_{\mu_2\alpha}$$
$$\tau^z(\boldsymbol{r}) = \sum_{\mu_1,\mu_2,\alpha} \varphi^*_{\mu_1}(\boldsymbol{r})\,\varphi_{\mu_2}(\boldsymbol{r})\,\alpha\,\psi^\dagger_{\mu_1\alpha}\psi_{\mu_2\alpha} . \quad (16)$$

Here we suppress the spin label (we assume full spin polarization); $\alpha = \pm 1$ is the valley label, and $\varphi_\mu(\boldsymbol{r})$ is the normalized symmetric gauge wavefunction of angular momentum μ in the lowest Landau level. We take $V(r) = e^2/\epsilon r$ and

$$W(\boldsymbol{r}-\boldsymbol{r}') = \left[W_0\ell^{-2}\,\delta(\boldsymbol{r}-\boldsymbol{r}') + W_1\nabla^2\delta(\boldsymbol{r}-\boldsymbol{r}')\right]\frac{e^2}{\epsilon\ell} . \quad (17)$$

W_0 and W_1 are phenomenological symmetry-breaking parameters. In the easy axis case, $W_1 < 0$, the symmetry breaking term leads to a gap $-\frac{2}{\pi}W_1\frac{e^2}{\epsilon\ell}$ in the dispersion relation for the valley waves [10]. W_0 is a local term that does not affect the form of the quasiparticles.

We now solve for the Hartree-Fock valley-textured quasiparticles, following the work of Fertig $et\ al.$ [11]. The Hartree-Fock wave functions for the skyrmion and anti-skyrmion are:

$$|\,\mathrm{sk}\,\rangle = \prod_{\mu=0}^{\infty}(u_\mu\psi^\dagger_{\mu+} + v_\mu\psi^\dagger_{\mu+1,-})\psi^\dagger_{0-}\,|\,0\,\rangle$$
$$|\,\overline{\mathrm{sk}}\,\rangle = \prod_{\mu=1}^{\infty}(u_\mu\psi^\dagger_{\mu+} + v_\mu\psi^\dagger_{\mu-1,-})\,|\,0\,\rangle , \quad (18)$$

where $|u_\mu|^2 + |v_\mu|^2 = 1$ and u_μ, v_μ are obtained by solving the Hartree-Fock equations self-consistently. Setting $u_\mu = 1$, we obtain the conventional quasiparticle and quasihole states $|\,\Psi_\pm\,\rangle$, with energies

$$E_- = -\frac{W_0}{2\pi}\frac{e^2}{\epsilon\ell}$$
$$E_+ = \left[\sqrt{\frac{\pi}{2}} - \frac{W_1}{\pi}\right]\frac{e^2}{\epsilon\ell} , \quad (19)$$
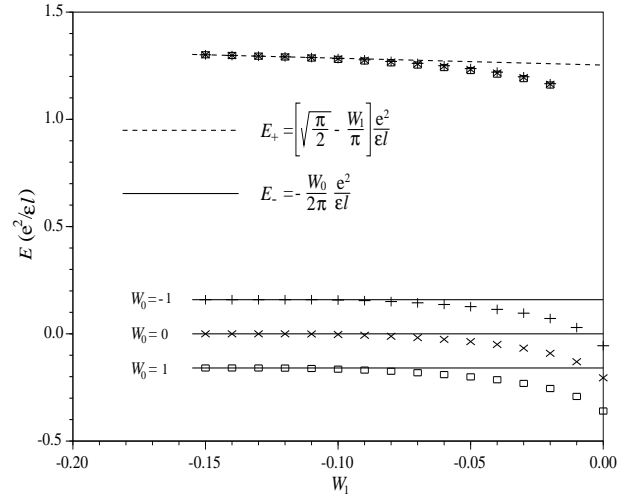

FIG. 1. Skyrmion (lower) and antiskyrmion (upper) energies $versus$ W_1 for three values of W_0. The solid and dashed lines correspond to the polarized quasiparticle energies of eq. 19.

relative to the energy of the polarized ground state $|\,\Psi_1[0]\,\rangle = \prod_{\mu=0}^{\infty}\psi^\dagger_{\mu+}\,|\,0\,\rangle$. Our Hartree-Fock results are plotted in figures 1 and 2. We find that valley skyrmions have lower energy than the polarized quasiparticles if $-0.117 \simeq W_{1c} < W_1 \leq 0$. (This is true for any W_0 as expected since this is a local term [6].)

$Discussion$ – To summarize, N component QH systems have an $SU(N)$ symmetry and $SU(N)$ skyrmions are the lowest energy charged excitations at $\nu = 1, 2, \ldots, N-1$ in the symmetric limit. $N > 2$ skyrmions may exist as valley skyrmions in (110) and (111) Si systems, as combined spin and valley skyrmions in standard (100) Si systems or in multi-layer QH systems. The most promising candidate probably being (110) or (111) SiGe systems.

What happens in a particular system depends crucially on the type and size of the symmetry breaking terms and experimental signatures will depend on the coupling to external probes. The energy gap to creating the lowest energy charged quasiparticles can be measured in activated transport, and for $SU(2)_{spin}$ skyrmions the spin can be obtained from such measurements by changing the in plane magnetic field. An external interfacial electric field plays a similar role for $SU(2)_{valley}$ skyrmions in Si, although in this case the energy depends on the field in a more complicated way. This can be used as a signal for $SU(2)_{valley}$ skyrmions and, in combination with an in plane magnetic field, to study crossovers between skyrmions with various quantum numbers in a limit where $\tilde{g}$, $-W_1$ and the valley Zeeman term are all small.
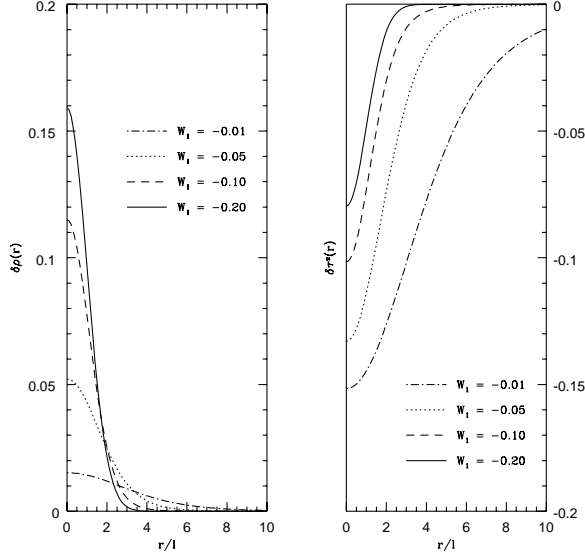
FIG. 2.   Skyrmion particle density, $\delta\rho$, and valley density, $\delta\tau^z$, for four values of W_1.

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